Low Torque Ripple Rotor Design of Interior Permanent Magnet Motor using Multi-phase Level-set and Phase-field Concept

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Abstract — This paper proposes a new optimization method to design the rotor of IPM motor consisted of permanent magnet(PM) and ferromagnetic material(FM) for reducing the torque ripple. To express three different material properties(PM, FM and air), the multi-phase level-set model representing two level-set functions is introduced and the concept of phase-field model is incorporated to control the complexity of structural boundaries. The optimization problem is formulated to minimize the torque ripple under the volume constraints of each material and two level-set functions are updated with respective sensitivity. To verify the usefulness of the proposed method, the rotor design example of IPM motor is performed and the novel configuration is obtained.

I. Introduction

Since interior permanent magnet(IPM) motor generates both magnetic and reluctance torque, it has high power density compared with the motor size. However, the reluctance torque also leads to high torque ripple which is the major factor of motor's noise and vibration, so optimizations for low torque ripple design of IPM motor have been performed with consideration of various schemes. Especially for the rotor design, previous studies show that the torque ripple can be minimized through the optimal shape of permanent magnet(PM) [1] or topological change of distribution of ferromagnetic material(FM) such as holes [2]. It is, however, difficult to expect various changes in the configuration through the previous optimization methods performed with a few parameters and a single material.

To obtain the innovative design of the rotor composed of different materials, the level-set based design method for magnetic field [3] is introduced with the multi-phase level-set model [4] in this paper. Four separated areas according to signs of two different level-set functions allow to express three material properties(PM,FM and air) in the rotor design. Each level-set function is moved to the optimal distribution under the design sensitivities for adjusting torques to the constant value for torque ripple reduction and volume constraints. In addition, free energy of phase-field model [5] is introduced in the distribution of level-set function and the objective function to control the complexity of final shape [6]. The coefficient of complexity performs an important role to simplify the structural boundaries to ensure the manufacturability. The design example of IPM motor is provided to investigate the usefulness of proposed method and achieve the optimal rotor design that delivers enhanced torque performance.

II. Problem Formulation

A. Multi-phase Level-set Model

The signed function is introduced in conventional level-set method to divide design domain into material and non-material region. To express more than two regions, the additional level-set function(ϕ_2) is employed to be independent of the other function(ϕ_1). With two different functions, four different regions can be defined as Fig. 1.

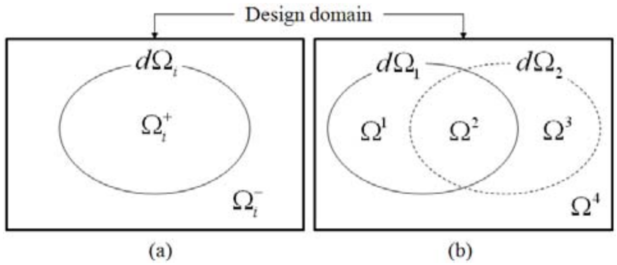

Fig. 1. Distribution of level-set function: (a)single-phase (b)multi-phase

To consider three material properties(PM,FM and air), region Ω^1 and Ω^2 represent PM, Ω^3 and Ω^4 represent FM and air, respectively. Since the Heaviside function($H(\phi)$) converts continuous level-set values to discrete form as 0 or 1, material property can be formulated to the following equation:

$$\begin{aligned} p(\phi_1,\phi_2) = & \; p_{PM} H(\phi_1)\left[H(\phi_2) + \{1-H(\phi_2)\}\right] \\ & + p_{FM}\{1-H(\phi_1)\} H(\phi_2) \\ & + p_{air}\{1-H(\phi_1)\}\{1-H(\phi_2)\} \end{aligned} \tag{1}$$

where the characteristic property p_i is set to the relative permeability(μ_r) of FM and remanent flux density(B_r) of PM for the analysis of the magnetic problem.

B. Phase-field Model

The phase-field model has two separated properties in each of the phases, with a smooth change in the interface where the interfacial free energy is occurred. Hence, the definition of the level-set function is modified to the following equation to express the interfacial region:

$$\begin{cases} 0 < \phi_i(\mathbf{x}) \le 1 & \Omega_i^+ \\ \phi_i(\mathbf{x}) = 0 & d\Omega_i \\ -1 \le \phi_i(\mathbf{x}) < 0 & \Omega_i^- \end{cases} \qquad i = 1,2 \tag{2}$$

The upper and lower bounds make the fictitious interface energy possible to be formulated by the term of the gradient of level-set function($\nabla\phi_i$) for boundary's simplification.

C. Optimization Problem

Adding fictitious interface energy to the objective functional($\overline{F}$), the optimization problem is formulated with volume constraint as follows:

$$\begin{aligned} &\min_{\phi_1,\phi_2}\ \overline{F}(\phi_1,\phi_2)=\left(\frac{T_j(\phi_1,\phi_2)}{T_0}-1\right)^2+\tau\int_\Omega \frac{1}{2}\left(\left|\nabla\phi_1\right|^2+\left|\nabla\phi_2\right|^2\right)d\Omega \\ &\text{subject to } G_1=\int_\Omega H(\phi_1)\,d\Omega \Big/ \int_\Omega d\Omega \le \mathrm{VF_{PM}} \\ &\qquad\qquad G_2=\int_\Omega \left(1-H(\phi_1)\right)H(\phi_2)\,d\Omega \Big/ \int_\Omega d\Omega \le \mathrm{VF_{FM}} \end{aligned} \tag{3}$$

where T_j stands for the torque at certain rotor position(j) and T_0 the constant target torque used for minimizing fluctuations of the torque. Also $\mathrm{VF_{PM}}$ and $\mathrm{VF_{FM}}$ are specified as volume fractions of PM and FM, respectively, and τ called the coefficient of complexity is a parameter representing the ratio of the fictitious interface energy and the objective function, which controls the complexity of the structural configuration. It is noted that two volume constraints are needed to deal with three different materials. The design sensitivity of each level-set function is calculated from the derivative of the objective functional and each material moves to the optimal distribution through following evolutional equation:

$$\frac{\partial\phi_i}{\partial t}=-\frac{\delta\overline{F}}{\delta\phi_i}=-\left[\frac{2\left(T_j-T_0\right)}{T_0^{\,2}}\cdot\frac{\partial T_j}{\partial\phi_i}-\tau\nabla^2\phi_i\right] \tag{4}$$

III. Design Example

The proposed method is applied to the rotor design of 12 pole-18 slot IPM motor. The reference design [7] with the average torque of 58.2Nm and the torque ripple of 91.3%, and the design domain are illustrated in Fig. 2. The optimization is progressed with the target torque of 65Nm under the same material usage as the reference design, PM of 20%, FM of 74% and air of 6% per rotor volume. The magnetization of PM is assigned to the radial direction from the center of rotor concerned with pole pairs.

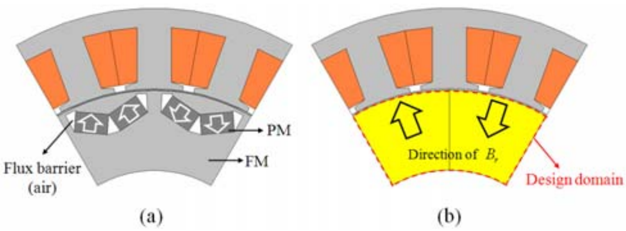

Fig. 2. Configuration of IPM motor: (a) reference design (b) design domain


Fig. 3. Optimal rotor design

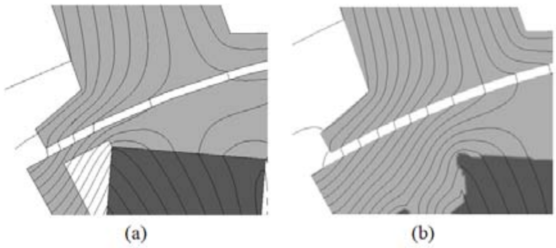

Fig. 4. Flux path of IPM motor: (a) reference design (b) optimal design

Fig. 3 depicts the optimal rotor design of IPM rotor consisted of PM represented by dark gray and FM by lighter color. Several flux barriers with air are generated inside FM to control the path of magnetic flux and reduce the leakage flux around PM for increasing the average torque. It is noted that the optimal shape of PM provides the uniformly distributed magnetic flux to the upper stator for preventing the ill-balanced tangential force, as shown in Fig. 4. Table I summarizes that the optimal configuration has the advantage of increasing the average torque to 12.0% and decreasing the torque ripple to 80.1% compared with the reference design.

TABLE I
COMPARISON BETWEEN REFERENCE AND OPTIMAL DESIGN

	Reference design	Optimal design
Average torque [Nm]	58.2	65.2
Torque ripple [%]	91.3	18.3

IV. References

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